>

EXAMEN_6 2025-2-F2

> restart

1)

> $Ecua := 2\cdot x\cdot y + y^2 + (x^2 + 2\cdot x\cdot y)\cdot y' = 0$

$$Ecua := 2\,x\,y(x) + y(x)^2 + \left(x^2 + 2\,x\,y(x)\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{1}$$

> with(DEtools) :

> odeadvisor(Ecua)

$$[[\_homogeneous, class\ A], \_exact, \_rational, [\_Abel, 2nd\ type, class\ B]] \tag{2}$$

Como exacta

> $M := 2\,x\,y + y^2$

$$M := 2\,x\,y + y^2 \tag{3}$$

> $N := (x^2 + 2\,x\,y)$

$$N := x^2 + 2\,x\,y \tag{4}$$

> Comprobar := diff(M, y) = diff(N, x)

$$Comprobar := 2\,x + 2\,y = 2\,x + 2\,y \tag{5}$$

> IntMx := expand(int(M, x))

$$IntMx := y\,x^2 + y^2\,x \tag{6}$$

> SolGral := IntMx + int((N − diff(IntMx, y)), y) = _C1

$$SolGral := y\,x^2 + y^2\,x = \_C1 \tag{7}$$

> $SolFinal := y(x)\cdot x^2 + y(x)^2\cdot x = \_C1$

$$SolFinal := y(x)\,x^2 + y(x)^2\,x = \_C1 \tag{8}$$

> DerSolFinal := isolate(diff(SolFinal, x), diff(y(x), x))

$$DerSolFinal := \frac{d}{dx}\,y(x) = \frac{-2\,x\,y(x) - y(x)^2}{x^2 + 2\,x\,y(x)} \tag{9}$$

> DerEcua := isolate(Ecua, diff(y(x), x))

$$DerEcua := \frac{d}{dx}\,y(x) = \frac{-2\,x\,y(x) - y(x)^2}{x^2 + 2\,x\,y(x)} \tag{10}$$

> ComprobarDos := rhs(DerSolFinal) − rhs(DerEcua) = 0

$$ComprobarDos := 0 = 0 \tag{11}$$

por coeficientes homogeneos

> Ecua

$$2\,x\,y(x) + y(x)^2 + \left(x^2 + 2\,x\,y(x)\right)\left(\frac{d}{dx}\,y(x)\right) = 0 \tag{12}$$

> EcuaDos := simplify(isolate(eval(subs(y(x) = u(x)·x, Ecua)), diff(u(x), x)))

$$EcuaDos := \frac{d}{dx}\,u(x) = -\frac{3\,u(x)\,(u(x) + 1)}{(1 + 2\,u(x))\,x} \tag{13}$$

> $SolGralDos := int\left(\frac{1}{x}, x\right) + int\left(\frac{1}{\left(\frac{3\,u\cdot(u+1)}{(1+2\,u)}\right)}, u\right) = \_C1$

$$SolGralDos := \ln(x) + \frac{\ln(u\,(u+1))}{3} = \_C1 \tag{14}$$

> $SolGralTres := \left(subs\left(u = \frac{y(x)}{x}, SolGralDos\right)\right)$

$$SolGralTres := \ln(x) + \frac{\ln\left(\frac{y(x)\left(\frac{y(x)}{x}+1\right)}{x}\right)}{3} = \_C1 \tag{15}$$

> $SolGralCuatro := expand(\,(simplify(\exp(lhs(SolGralTres)))^3 = \_C1)\,)$

$$SolGralCuatro := y(x)\,x^2 + y(x)^2\,x = \_C1 \tag{16}$$

> $SolFinal$

$$y(x)\,x^2 + y(x)^2\,x = \_C1 \tag{17}$$

> $restart$

2)

> $Ecua := r\cdot diff(\theta(r), r) = \frac{(r^2+a^2)}{r^2}$

$$Ecua := r\left(\frac{d}{dr}\,\theta(r)\right) = \frac{a^2+r^2}{r^2} \tag{18}$$

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_quadrature] \tag{19}$$

> $IntFact := intfactor(Ecua)$

$$IntFact := \frac{1}{r} \tag{20}$$

> $M := -rhs(Ecua)$

$$M := -\frac{a^2+r^2}{r^2} \tag{21}$$

> $N := r$

$$N := r \tag{22}$$

> $MM := IntFact\cdot M$

$$MM := -\frac{a^2+r^2}{r^3} \tag{23}$$

> $NN := IntFact\cdot N$

$$NN := 1 \tag{24}$$

> $Comprobar := diff(MM, \theta) = diff(NN, r)$

$$Comprobar := 0 = 0 \tag{25}$$

> $IntNNtheta := int(NN, \theta)$

$$IntNNtheta := \theta \quad (26)$$

> $SolGral := IntNNtheta + int((MM - diff(IntNNtheta, r)), r) = \_C1$

$$SolGral := \theta + \frac{a^2}{2 r^2} - \ln(r) = \_C1 \quad (27)$$

> $SolFinal := isolate(SolGral, \text{theta})$

$$SolFinal := \theta = \_C1 - \frac{a^2}{2 r^2} + \ln(r) \quad (28)$$

> $SolFinalDos := \text{theta}(r) = rhs(SolFinal)$

$$SolFinalDos := \theta(r) = \_C1 - \frac{a^2}{2 r^2} + \ln(r) \quad (29)$$

> $Ecua$

$$r \left(\frac{d}{dr} \theta(r) \right) = \frac{a^2 + r^2}{r^2} \quad (30)$$

> $ComprobarDos := simplify(eval(subs(\text{theta}(r) = rhs(SolFinalDos), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarDos := 0 = 0 \quad (31)$$

> $restart$

3)

> $Ecua := y'' - 2 \cdot y' + y = x \cdot \sinh(x)$

$$Ecua := \frac{d^2}{dx^2} y(x) - 2 \frac{d}{dx} y(x) + y(x) = x \sinh(x) \quad (32)$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) - 2 \frac{d}{dx} y(x) + y(x) = 0 \quad (33)$$

> $Q := rhs(Ecua)$

$$Q := x \sinh(x) \quad (34)$$

> $EcuaCarac := m^2 - 2 \cdot m + 1 = 0$

$$EcuaCarac := m^2 - 2 m + 1 = 0 \quad (35)$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 1, 1 \quad (36)$$

> $yy[1] := \exp(Raiz[1] \cdot x); yy[2] := x \cdot \exp(Raiz[1] \cdot x)$

$$yy_1 := e^x$$

$$yy_2 := x e^x \quad (37)$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & x e^x \\ e^x & e^x + x e^x \end{bmatrix} \quad (38)$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & x\sinh(x) \end{bmatrix} \quad (39)$$

> $Para := linsolve(WW, BB)$

$$Para := \begin{bmatrix} -\frac{x^2 \sinh(x)}{e^x} & \frac{x \sinh(x)}{e^x} \end{bmatrix} \quad (40)$$

> $Aprima := Para[1]; Bprima := Para[2]$

$$Aprima := -\frac{x^2 \sinh(x)}{e^x}$$

$$Bprima := \frac{x \sinh(x)}{e^x} \quad (41)$$

> $SolGral := y(x) = expand(simplify((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2]))$

$$SolGral := y(x) = -\frac{1}{8\,e^x} - \frac{x}{8\,e^x} + \frac{e^x x^3}{12} + x\,e^x\,\_C2 + e^x\,\_C1 \quad (42)$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGral), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (43)$$

> restart

4)

> $H := \frac{(s^2 + 2 \cdot s + 5)}{(s^2 - 6 \cdot s + 10)}$

$$H := \frac{s^2 + 2\,s + 5}{s^2 - 6\,s + 10} \quad (44)$$

> with(inttrans) :

> $h := expand(invlaplace(H, s, t))$

$$h := \mathrm{Dirac}(t) + 8\,(e^t)^3 \cos(t) + 19\,(e^t)^3 \sin(t) \quad (45)$$

> restart

5)

> $Ecua := diff(y(t), t\$2) + 4 \cdot y(t) = \cos(t - \mathrm{Pi}) \cdot \mathrm{Heaviside}(t - \mathrm{Pi})$

$$Ecua := \frac{d^2}{dt^2}\,y(t) + 4\,y(t) = -\cos(t)\,\mathrm{Heaviside}(t - \pi) \quad (46)$$

> $CondIni := y(0) = 0, D(y)(0) = 1$

$$CondIni := y(0) = 0, D(y)(0) = 1 \quad (47)$$

> with(inttrans) :

> $EcuaTransLap := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTransLap := s^2\,\mathcal{L}(y(t), t, s) - 1 + 4\,\mathcal{L}(y(t), t, s) = \frac{e^{-s\pi}\,s}{s^2 + 1} \quad (48)$$

> $SolTransLap := simplify(isolate(EcuaTransLap, laplace(y(t), t, s)))$

(49)

$$SolTransLap := \mathcal{L}(y(t), t, s) = \frac{e^{-s\pi} s + s^2 + 1}{(s^2+1)(s^2+4)} \quad \textbf{(49)}$$

> $SolPart := invlaplace(SolTransLap, s, t)$

$$SolPart := y(t) = \frac{\sin(2t)}{2} - \frac{\text{Heaviside}(t-\pi)(\cos(t)+\cos(2t))}{3} \quad \textbf{(50)}$$

> $ComprobarUno := simplify(subs(t=0, SolPart))$

$$ComprobarUno := y(0) = 0 \quad \textbf{(51)}$$

> $ComprobarDos := D(y)(0) = simplify(subs(t=0, rhs(diff(SolPart, t))))$

$$ComprobarDos := D(y)(0) = 1 \quad \textbf{(52)}$$

> $CondIni$

$$y(0) = 0, D(y)(0) = 1 \quad \textbf{(53)}$$

> $ComprobarTres := simplify(eval(subs(y(t) = rhs(SolPart), lhs(Ecua) - rhs(Ecua) = 0)))$

$$ComprobarTres := 0 = 0 \quad \textbf{(54)}$$

> $restart$

6)

> $Ecua := diff(y(t), t\$3) - 2\cdot diff(y(t), t\$2) + y(t) = \cos(t)$

$$Ecua := \frac{d^3}{dt^3} y(t) - 2 \frac{d^2}{dt^2} y(t) + y(t) = \cos(t) \quad \textbf{(55)}$$

> $CondIni := y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 2$

$$CondIni := y(0) = 1, D(y)(0) = 0, D^{(2)}(y)(0) = 2 \quad \textbf{(56)}$$

a) convertir a un sistema

> $FuncUno := y(t) = yy[1](t)$

$$FuncUno := y(t) = yy_1(t) \quad \textbf{(57)}$$

> $diff(y(t), t) := diff(yy[1](t), t) = yy[2](t)$

$$\frac{d}{dt} y(t) := \frac{d}{dt} yy_1(t) = yy_2(t) \quad \textbf{(58)}$$

> $diff(y(t), t\$2) := diff(yy[2](t), t) = yy[3](t)$

$$\frac{d^2}{dt^2} y(t) := \frac{d}{dt} yy_2(t) = yy_3(t) \quad \textbf{(59)}$$

> $diff(y(t), t\$3) := diff(yy[3](t), t) = -yy[1](t) + 2\cdot yy[3](t) + \cos(t)$

$$\frac{d^3}{dt^3} y(t) := \frac{d}{dt} yy_3(t) = -yy_1(t) + 2\, yy_3(t) + \cos(t) \quad \textbf{(60)}$$

> $Ecua$

$$\frac{d^3}{dt^3} y(t) - 2 \frac{d^2}{dt^2} y(t) + y(t) = \cos(t) \quad \textbf{(61)}$$

> $Sistema := ([diff(yy[1](t), t) = yy[2](t), diff(yy[2](t), t) = yy[3](t), diff(yy[3](t), t) = -yy[1](t) + 2\cdot yy[3](t) + \cos(t)]) : Sistema[1]; Sistema[2]; Sistema[3]$

$$\frac{d}{dt} yy_1(t) = yy_2(t)$$

$$\frac{d}{dt} yy_2(t) = yy_3(t)$$

$$\frac{d}{dt} yy_3(t) = -yy_1(t) + 2\, yy_3(t) + \cos(t) \quad \textbf{(62)}$$

> $CondIniDos := yy[1](0) = 1, yy[2](0) = 0, yy[3](0) = 2$

$$CondIniDos := yy_1(0) = 1, yy_2(0) = 0, yy_3(0) = 2 \quad \textbf{(63)}$$

b) resolver con Transformada de Laplace

> $with(inttrans):$

> $SistTransdLap[1] := subs(CondIniDos, laplace(Sistema[1], t, s))$

$$SistTransdLap_1 := s\,\mathcal{L}\left(yy_1(t), t, s\right) - 1 = \mathcal{L}\left(yy_2(t), t, s\right) \quad \textbf{(64)}$$

> $SistTransdLap[2] := subs(CondIniDos, laplace(Sistema[2], t, s))$

$$SistTransdLap_2 := s\,\mathcal{L}\left(yy_2(t), t, s\right) = \mathcal{L}\left(yy_3(t), t, s\right) \quad \textbf{(65)}$$

> $SistTransdLap[3] := subs(CondIniDos, laplace(Sistema[3], t, s))$

$$SistTransdLap_3 := s\,\mathcal{L}\left(yy_3(t), t, s\right) - 2 = -\mathcal{L}\left(yy_1(t), t, s\right) + 2\,\mathcal{L}\left(yy_3(t), t, s\right) + \frac{s}{s^2+1} \quad \textbf{(66)}$$

> $SistemaDos := SistTransdLap[1], SistTransdLap[2], SistTransdLap[3];$

$$SistemaDos := s\,\mathcal{L}\left(yy_1(t), t, s\right) - 1 = \mathcal{L}\left(yy_2(t), t, s\right), s\,\mathcal{L}\left(yy_2(t), t, s\right) = \mathcal{L}\left(yy_3(t), t, s\right), \quad \textbf{(67)}$$

$$s\,\mathcal{L}\left(yy_3(t), t, s\right) - 2 = -\mathcal{L}\left(yy_1(t), t, s\right) + 2\,\mathcal{L}\left(yy_3(t), t, s\right) + \frac{s}{s^2+1}$$

> $with(linalg):$

> $SolTransLap := solve([SistemaDos], [laplace(yy[1](t), t, s), laplace(yy[2](t), t, s), laplace(yy[3](t), t, s)])$

$$SolTransLap := \left[\left[\mathcal{L}\left(yy_1(t), t, s\right) = \frac{s^4 - 2s^3 + 3s^2 - s + 2}{s^5 - 2s^4 + s^3 - s^2 + 1}, \mathcal{L}\left(yy_2(t), t, s\right) = \frac{2s^3 + 2s - 1}{s^5 - 2s^4 + s^3 - s^2 + 1}, \mathcal{L}\left(yy_3(t), t, s\right) = \frac{s\,(2s^3 + 2s - 1)}{s^5 - 2s^4 + s^3 - s^2 + 1}\right]\right] \quad \textbf{(68)}$$

> $SolPart[1] := simplify(invlaplace(SolTransLap[1, 1], s, t))$

$$SolPart_1 := yy_1(t) = \frac{e^{\frac{t}{2}} \sinh\left(\frac{t\sqrt{5}}{2}\right)\sqrt{5}}{5} + \frac{11\, e^{\frac{t}{2}} \cosh\left(\frac{t\sqrt{5}}{2}\right)}{5} - \frac{3\, e^{t}}{2} + \frac{3\cos(t)}{10} - \frac{\sin(t)}{10} \quad \textbf{(69)}$$

> $SolPart[2] := simplify(invlaplace(SolTransLap[1, 2], s, t))$

$$SolPart_2 := yy_2(t) = \frac{6\, e^{\frac{t}{2}} \sinh\left(\frac{t\sqrt{5}}{2}\right)\sqrt{5}}{5} + \frac{8\, e^{\frac{t}{2}} \cosh\left(\frac{t\sqrt{5}}{2}\right)}{5} - \frac{3\, e^{t}}{2} - \frac{\cos(t)}{10} - \frac{3\sin(t)}{10} \quad \textbf{(70)}$$

> $SolPart[3] := simplify(invlaplace(SolTransLap[1, 3], s, t))$

$$SolPart_3 := yy_3(t) = \frac{7\, e^{\frac{t}{2}} \sinh\left(\frac{t\sqrt{5}}{2}\right)\sqrt{5}}{5} + \frac{19\, e^{\frac{t}{2}} \cosh\left(\frac{t\sqrt{5}}{2}\right)}{5} - \frac{3\, e^{t}}{2} - \frac{3\cos(t)}{10} + \frac{\sin(t)}{10} \quad \textbf{(71)}$$

> $SolFinal := y(t) = rhs(SolPart[1])$

$$SolFinal := y(t) = \frac{e^{\frac{t}{2}} \sinh\left(\frac{t\sqrt{5}}{2}\right)\sqrt{5}}{5} + \frac{11\, e^{\frac{t}{2}} \cosh\left(\frac{t\sqrt{5}}{2}\right)}{5} - \frac{3\, e^{t}}{2} + \frac{3\cos(t)}{10} - \frac{\sin(t)}{10} \quad \textbf{(72)}$$

> $restart$

7)

> $Ecua := diff(u(x, t), x\$2) = diff(u(x, t), t) + 2 \cdot u(x, t)$

$$Ecua := \frac{\partial^2}{\partial x^2} u(x, t) = \frac{\partial}{\partial t} u(x, t) + 2\, u(x, t) \quad \textbf{(73)}$$

> $EcuaDos := eval(subs(u(x, t) = F(x) \cdot G(t), Ecua))$

$$EcuaDos := \left(\frac{d^2}{dx^2} F(x)\right) G(t) = F(x) \left(\frac{d}{dt} G(t)\right) + 2\, F(x)\, G(t) \quad \textbf{(74)}$$

> $EcuaSep := \frac{lhs(EcuaDos)}{F(x) \cdot G(t)} = simplify\left(\frac{rhs(EcuaDos)}{F(x) \cdot G(t)}\right)$

$$EcuaSep := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{\frac{d}{dt} G(t) + 2\, G(t)}{G(t)} \quad \textbf{(75)}$$

a) para cte sep igual a 4

> $EcuaPosX := lhs(EcuaSep) = 4$

$$EcuaPosX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = 4 \quad \textbf{(76)}$$

> $EcuaPosT := rhs(EcuaSep) = 4$

$$EcuaPosT := \frac{\frac{d}{dt} G(t) + 2\, G(t)}{G(t)} = 4 \quad \textbf{(77)}$$

> $SolPosX := dsolve(EcuaPosX)$

$$SolPosX := F(x) = c_1\, e^{-2x} + c_2\, e^{2x} \quad \textbf{(78)}$$

> $SolPosT := dsolve(EcuaPosT)$

$$SolPosT := G(t) = c_1\, e^{2t} \quad \textbf{(79)}$$

> $SolPosGral := u(x, t) = rhs(SolPosX) \cdot subs(c_1 = 1, rhs(SolPosT))$

(80)

$$SolPosGral := u(x,t) = \left(c_1 e^{-2x} + c_2 e^{2x}\right) e^{2t} \quad (80)$$

para cte sep igual a -4

> $EcuaNegX := lhs(EcuaSep) = -4$

$$EcuaNegX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = -4 \quad (81)$$

> $EcuaNegT := rhs(EcuaSep) = -4$

$$EcuaNegT := \frac{\frac{d}{dt} G(t) + 2\, G(t)}{G(t)} = -4 \quad (82)$$

> $SolNegX := dsolve(EcuaNegX)$

$$SolNegX := F(x) = c_1 \sin(2x) + c_2 \cos(2x) \quad (83)$$

> $SolNegT := dsolve(EcuaNegT)$

$$SolNegT := G(t) = c_1 e^{-6t} \quad (84)$$

> $SolNegGral := u(x,t) = rhs(SolNegX) \cdot subs\left(c_1 = 1, rhs(SolNegT)\right)$

$$SolNegGral := u(x,t) = \left(c_1 \sin(2x) + c_2 \cos(2x)\right) e^{-6t} \quad (85)$$

> $restart$

8)

> $f := x^2 + x$

$$f := x^2 + x \quad (86)$$

> $L := \frac{\mathrm{Pi}}{2}$

$$L := \frac{\pi}{2} \quad (87)$$

> $a[0] := \frac{1}{L} \cdot int(f, x = -L..L)$

$$a_0 := \frac{\pi^2}{6} \quad (88)$$

> $a[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

$$a_n := \frac{(-1)^n}{n^2} \quad (89)$$

> $b[n] := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^n, \frac{1}{L} \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), x = -L..L\right)\right)$

$$b_n := -\frac{(-1)^n}{n} \quad (90)$$

> $STF5 := \frac{a[0]}{2} + sum\left(a[n] \cdot \cos\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right) + b[n] \cdot \sin\left(\frac{n \cdot \mathrm{Pi}}{L} \cdot x\right), n = 1..5\right)$

$$STF5 := \frac{\pi^2}{12} - \cos(2x) + \sin(2x) + \frac{\cos(4x)}{4} - \frac{\sin(4x)}{2} - \frac{\cos(6x)}{9} + \frac{\sin(6x)}{3} + \frac{\cos(8x)}{16} - \frac{\sin(8x)}{4} - \frac{\cos(10x)}{25} + \frac{\sin(10x)}{5} \qquad \textbf{(91)}$$

> *restart*

FIN EXAMEN