

Examen Ecuaciones Diferenciales

FINAL

Grupo 13

Semestre 2026-1

SEGUNDA VUELTA

> restart

1a) Resuelva el problema de valor inicial

> $Ecua := y' = \frac{(x \cdot y^2 - 1)}{1 - x^2 \cdot y}$

$$Ecua := \frac{d}{dx} y(x) = \frac{x y(x)^2 - 1}{1 - x^2 y(x)} \quad (1)$$

> $CondIni := y(0) = 1$

$$CondIni := y(0) = 1 \quad (2)$$

RESPUESTA

> $EcuaDos := -rhs(Ecua) + lhs(Ecua) = 0$

$$EcuaDos := -\frac{x y(x)^2 - 1}{1 - x^2 y(x)} + \frac{d}{dx} y(x) = 0 \quad (3)$$

> $EcuaTres := -x y(x)^2 + 1 + (1 - x^2 y(x)) \cdot diff(y(x), x) = 0$

$$EcuaTres := -x y(x)^2 + 1 + (1 - x^2 y(x)) \left(\frac{d}{dx} y(x) \right) = 0 \quad (4)$$

> with(DEtools):

> odeadvisor(EcuaTres)

[_exact, _rational, [_Abel, 2nd type, class B]] (5)

> $M := -x y^2 + 1$

$$M := -x y^2 + 1 \quad (6)$$

> $N := 1 - x^2 y$

$$N := -x^2 y + 1 \quad (7)$$

> $IntMx := int(M, x)$

$$IntMx := -\frac{1}{2} x^2 y^2 + x \quad (8)$$

> $SolGral := IntMx + int((N - diff(IntMx, y)), y) = _CI$

$$SolGral := -\frac{1}{2} x^2 y^2 + x + y = _CI \quad (9)$$

> $SolGralFinal := -\frac{1}{2} y(x)^2 x^2 + x + y(x) = _CI$

$$SolGralFinal := -\frac{y(x)^2 x^2}{2} + x + y(x) = _CI \quad (10)$$

$$\begin{aligned} &> \text{DerSolFinal} := \text{isolate}(\text{diff}(\text{SolGralFinal}, x), \text{diff}(y(x), x)) \\ &\quad \text{DerSolFinal} := \frac{d}{dx} y(x) = \frac{x y(x)^2 - 1}{1 - x^2 y(x)} \end{aligned} \quad (11)$$

$$\begin{aligned} &> \text{Ecua} \\ &\quad \frac{d}{dx} y(x) = \frac{x y(x)^2 - 1}{1 - x^2 y(x)} \end{aligned} \quad (12)$$

$$\begin{aligned} &> \text{Comprobar} := \text{rhs}(\text{Ecua}) - \text{rhs}(\text{DerSolFinal}) = 0 \\ &\quad \text{Comprobar} := 0 = 0 \end{aligned} \quad (13)$$

$$\begin{aligned} &> \text{Parametro} := \text{subs}(y(x) = 1, x = 0, \text{SolGralFinal}) \\ &\quad \text{Parametro} := 1 = _C1 \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{SolPart} := \text{subs}(_C1 = \text{lhs}(\text{Parametro}), \text{SolGralFinal}) \\ &\quad \text{SolPart} := -\frac{y(x)^2 x^2}{2} + x + y(x) = 1 \end{aligned} \quad (15)$$

> restart

2a) Resuelva la ecuación diferencial

$$\begin{aligned} &> \text{Ecua} := y'' + y = 3 \cdot \sec(x) + 1 \\ &\quad \text{Ecua} := \frac{d^2}{dx^2} y(x) + y(x) = 3 \sec(x) + 1 \end{aligned} \quad (16)$$

> RESPUESTA

$$\begin{aligned} &> \text{EcuaHom} := \text{lhs}(\text{Ecua}) = 0 \\ &\quad \text{EcuaHom} := \frac{d^2}{dx^2} y(x) + y(x) = 0 \end{aligned} \quad (17)$$

$$\begin{aligned} &> Q := \text{rhs}(\text{Ecua}) \\ &\quad Q := 3 \sec(x) + 1 \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{EcuaCarac} := m^2 + 1 = 0 \\ &\quad \text{EcuaCarac} := m^2 + 1 = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{Raiz} := \text{solve}(\text{EcuaCarac}) \\ &\quad \text{Raiz} := I, -I \end{aligned} \quad (20)$$

$$\begin{aligned} &> yy[1] := \cos(\text{Im}(\text{Raiz}[1]) \cdot x); yy[2] := \sin(\text{Im}(\text{Raiz}[1]) \cdot x) \\ &\quad yy_1 := \cos(x) \\ &\quad yy_2 := \sin(x) \end{aligned} \quad (21)$$

> with(linalg) :

$$\begin{aligned} &> WW := \text{wronskian}([yy[1], yy[2]], x) \\ &\quad WW := \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \end{aligned} \quad (22)$$

$$\begin{aligned} &> BB := \text{array}([0, Q]) \\ &\quad BB := \begin{bmatrix} 0 & 3 \sec(x) + 1 \end{bmatrix} \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{Para} := \text{simplify}(\text{linsolve}(\text{WW}, \text{BB})) \\ &\quad \text{Para} := \begin{bmatrix} -3 \tan(x) - \sin(x) & 3 + \cos(x) \end{bmatrix} \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{Aprima} := \text{Para}[1]; \text{Bprima} := \text{Para}[2] \\ &\quad \text{Aprima} := -3 \tan(x) - \sin(x) \\ &\quad \text{Bprima} := 3 + \cos(x) \end{aligned} \quad (25)$$

$$\begin{aligned} &> \text{SolGral} := y(x) = \text{expand}((\text{int}(\text{Aprima}, x) + _C1) \cdot \text{yy}[1] + (\text{int}(\text{Bprima}, x) + _C2) \cdot \text{yy}[2]) \\ &\quad \text{SolGral} := y(x) = \cos(x)^2 + 3 \cos(x) \ln(\cos(x)) + \cos(x) _C1 + 3 \sin(x) x + \sin(x)^2 \\ &\quad \quad + \sin(x) _C2 \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{SolFinal} := \text{simplify}(\text{SolGral}) \\ &\quad \text{SolFinal} := y(x) = (_C2 + 3 x) \sin(x) + 1 + 3 \cos(x) \ln(\cos(x)) + \cos(x) _C1 \end{aligned} \quad (27)$$

$$\begin{aligned} &> \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolGral}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ &\quad \text{Comprobar} := 0 = 0 \end{aligned} \quad (28)$$

> restart

3a) Resolver

$$\begin{aligned} &> \text{Ecua} := \text{diff}(y(t), t\$2) + y(t) = 4 \cdot \text{Dirac}(t - 2 \cdot \text{Pi}) \\ &\quad \text{Ecua} := \frac{d^2}{dt^2} y(t) + y(t) = 4 \text{Dirac}(t - 2 \pi) \end{aligned} \quad (29)$$

$$\begin{aligned} &> \text{CondIni} := y(0) = 1, \text{D}(y)(0) = 1 \\ &\quad \text{CondIni} := y(0) = 1, \text{D}(y)(0) = 1 \end{aligned} \quad (30)$$

RESPUESTA

$$\begin{aligned} &> \text{with}(\text{inttrans}) : \\ &> \text{EcuaTL} := \text{subs}(\text{CondIni}, \text{laplace}(\text{Ecua}, t, s)) \\ &\quad \text{EcuaTL} := s^2 \mathcal{L}(y(t), t, s) - 1 - s + \mathcal{L}(y(t), t, s) = 4 e^{-2 s \pi} \end{aligned} \quad (31)$$

$$\begin{aligned} &> \text{SolTL} := \text{isolate}(\text{EcuaTL}, \text{laplace}(y(t), t, s)) \\ &\quad \text{SolTL} := \mathcal{L}(y(t), t, s) = \frac{4 e^{-2 s \pi} + s + 1}{s^2 + 1} \end{aligned} \quad (32)$$

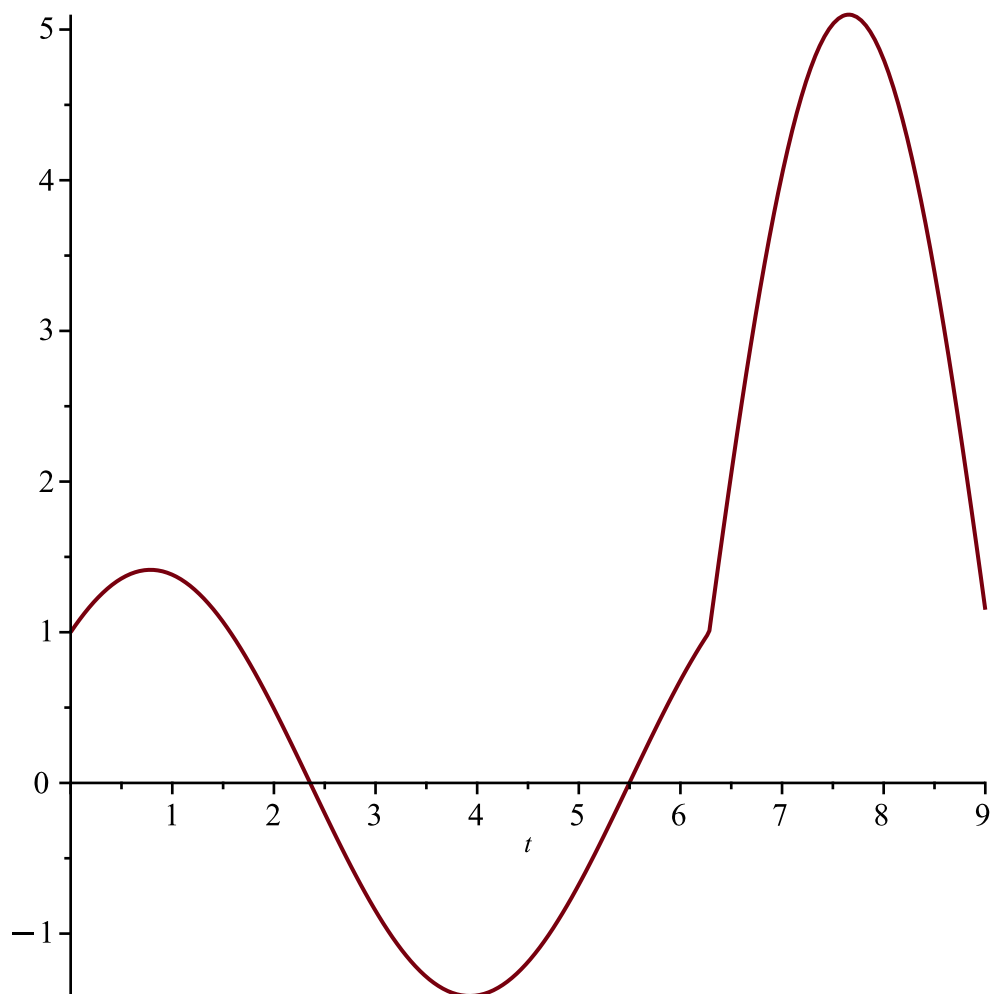
$$\begin{aligned} &> \text{SolPart} := \text{invlaplace}(\text{SolTL}, s, t) \\ &\quad \text{SolPart} := y(t) = \cos(t) + \sin(t) (1 + 4 \text{Heaviside}(t - 2 \pi)) \end{aligned} \quad (33)$$

$$\begin{aligned} &> \text{CondIniUno} := \text{simplify}(\text{subs}(t = 0, \text{SolPart})) \\ &\quad \text{CondIniUno} := y(0) = 1 \end{aligned} \quad (34)$$

$$\begin{aligned} &> \text{CondIniDos} := \text{D}(y)(0) = \text{simplify}(\text{rhs}(\text{subs}(t = 0, \text{diff}(\text{SolPart}, t)))) \\ &\quad \text{CondIniDos} := \text{D}(y)(0) = 1 \end{aligned} \quad (35)$$

$$\begin{aligned} &> \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(y(t) = \text{rhs}(\text{SolPart}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ &\quad \text{Comprobar} := 0 = 0 \end{aligned} \quad (36)$$

$$> \text{plot}(\text{rhs}(\text{SolPart}), t = 0 .. 9)$$



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> restart
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4a) Determine el sistema equivalente y resuélvalo de

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> Ecua := y'' + 4·y' + 4·y = 8
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$$Ecua := \frac{d^2}{dx^2} y(x) + 4 \frac{d}{dx} y(x) + 4 y(x) = 8 \quad (37)$$

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>
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RESPUESTA

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>
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> Sistema := diff(y[1](x), x) = y[2](x), diff(y[2](x), x) = -4·y[1](x) - 4·y[2](x) + 8 :
Sistema[1]; Sistema[2]
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$$\frac{d}{dx} y_1(x) = y_2(x)$$

$$\frac{d}{dx} y_2(x) = -4 y_1(x) - 4 y_2(x) + 8 \quad (38)$$

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> AA := array([ [0, 1], [-4, -4] ])
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$$AA := \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix} \quad (39)$$

> $Xcero := array([_C1, _C2])$

$$Xcero := \begin{bmatrix} _C1 & _C2 \end{bmatrix} \quad (40)$$

> $BB := array([0, 8])$

$$BB := \begin{bmatrix} 0 & 8 \end{bmatrix} \quad (41)$$

> $with(linalg) :$

> $MatExp := exponential(AA, x)$

$$MatExp := \begin{bmatrix} e^{-2x} + 2x e^{-2x} & x e^{-2x} \\ -4x e^{-2x} & e^{-2x} - 2x e^{-2x} \end{bmatrix} \quad (42)$$

> $SolHom := evalm(MatExp \&* Xcero) : y[1](x) = simplify(SolHom[1]); y[2](x) = simplify(SolHom[2])$

$$\begin{aligned} y_1(x) &= e^{-2x} (2x _C1 + _C2 x + _C1) \\ y_2(x) &= ((-2x + 1) _C2 - 4x _C1) e^{-2x} \end{aligned} \quad (43)$$

> $ComprobarUno := simplify(subs(x=0, y[1](x) = SolHom[1]))$

$$ComprobarUno := y_1(0) = _C1 \quad (44)$$

> $ComprobarDos := simplify(subs(x=0, y[2](x) = SolHom[2]))$

$$ComprobarDos := y_2(0) = _C2 \quad (45)$$

> $MatExpTau := map(rcurry(eval, x = x - tau'), MatExp)$

$$MatExpTau := \begin{bmatrix} e^{-2x+2\tau} + 2(x-\tau) e^{-2x+2\tau} & (x-\tau) e^{-2x+2\tau} \\ -4(x-\tau) e^{-2x+2\tau} & e^{-2x+2\tau} - 2(x-\tau) e^{-2x+2\tau} \end{bmatrix} \quad (46)$$

> $ProdTau := evalm(MatExpTau \&* BB)$

$$ProdTau := \begin{bmatrix} 8(x-\tau) e^{-2x+2\tau} & 8e^{-2x+2\tau} - 16(x-\tau) e^{-2x+2\tau} \end{bmatrix} \quad (47)$$

> $SolNoHom := simplify(map(int, ProdTau, tau = 0..x)) : y[1](x) = SolNoHom[1]; y[2](x) = SolNoHom[2]$

$$\begin{aligned} y_1(x) &= 2 + (-4x - 2) e^{-2x} \\ y_2(x) &= 8x e^{-2x} \end{aligned} \quad (48)$$

> $ComprobarCinco := simplify(subs(x=0, y[1](x) = SolNoHom[1]))$

$$ComprobarCinco := y_1(0) = 0 \quad (49)$$

> $ComprobarSeis := simplify(subs(x=0, y[2](x) = SolNoHom[2]))$

$$ComprobarSeis := y_2(0) = 0 \quad (50)$$

> $SolFinal[1] := y[1](x) = simplify(SolHom[1] + SolNoHom[1]); SolFinal[2] := y[2](x) = simplify(SolHom[2] + SolNoHom[2])$

$$\begin{aligned} SolFinal_1 &:= y_1(x) = 2 + ((2_C1 + _C2 - 4)x + _C1 - 2) e^{-2x} \\ SolFinal_2 &:= y_2(x) = e^{-2x} (-4x _C1 - 2_C2 x + _C2 + 8x) \end{aligned} \quad (51)$$

> Sistema[1]

$$\frac{d}{dx} y_1(x) = y_2(x) \quad (52)$$

> ComprobarTres := simplify(eval(subs(y[1](x) = rhs(SolFinal[1]), y[2](x) = rhs(SolFinal[2]), lhs(Sistema[1]) - rhs(Sistema[1]) = 0)))
ComprobarTres := 0 = 0

(53)

> Sistema[2]

$$\frac{d}{dx} y_2(x) = -4 y_1(x) - 4 y_2(x) + 8 \quad (54)$$

> ComprobarCuatro := simplify(eval(subs(y[1](x) = rhs(SolFinal[1]), y[2](x) = rhs(SolFinal[2]), lhs(Sistema[2]) - rhs(Sistema[2]) = 0)))
ComprobarCuatro := 0 = 0

(55)

> restart

5a) Obtener la solución completa para constante positiva

> Ecua := diff(u(x, t), x\$2) + diff(u(x, t), x, t) = 4 * t * diff(u(x, t), x)

$$Ecua := \frac{\partial^2}{\partial x^2} u(x, t) + \frac{\partial^2}{\partial t \partial x} u(x, t) = 4 t \left(\frac{\partial}{\partial x} u(x, t) \right) \quad (56)$$

>

RESPUESTA

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> EcuaSeparable := eval(subs(u(x, t) = F(x) * G(t), Ecua))

$$EcuaSeparable := \left(\frac{d^2}{dx^2} F(x) \right) G(t) + \left(\frac{d}{dx} F(x) \right) \left(\frac{d}{dt} G(t) \right) = 4 t \left(\frac{d}{dx} F(x) \right) G(t) \quad (57)$$

> EcuaSeparada :=
$$\frac{\left(lhs(EcuaSeparable) - \left(\frac{d}{dx} F(x) \right) \left(\frac{d}{dt} G(t) \right) \right)}{\frac{d}{dx} F(x) \cdot G(t)}$$

$$= simplify \left(\frac{\left(rhs(EcuaSeparable) - \left(\frac{d}{dx} F(x) \right) \left(\frac{d}{dt} G(t) \right) \right)}{\frac{d}{dx} F(x) \cdot G(t)} \right)$$

$$EcuaSeparada := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \frac{4 G(t) t - \frac{d}{dt} G(t)}{G(t)} \quad (58)$$

> EcuaX := lhs(EcuaSeparada) = β^2

$$EcuaX := \frac{\frac{d^2}{dx^2} F(x)}{\frac{d}{dx} F(x)} = \beta^2 \quad (59)$$

$$\begin{aligned} &> \text{EcuaT} := \text{rhs}(\text{EcuaSeparada}) = \beta^2 \\ & \text{EcuaT} := \frac{4 G(t) t - \frac{d}{dt} G(t)}{G(t)} = \beta^2 \end{aligned} \quad (60)$$

$$\begin{aligned} &> \text{SolX} := \text{dsolve}(\text{EcuaX}) \\ & \text{SolX} := F(x) = c_1 + c_2 e^{\beta^2 x} \end{aligned} \quad (61)$$

$$\begin{aligned} &> \text{SolT} := \text{dsolve}(\text{EcuaT}) \\ & \text{SolT} := G(t) = c_1 e^{-t(\beta^2 - 2)} \end{aligned} \quad (62)$$

$$\begin{aligned} &> \text{SolGral} := u(x, t) = \text{rhs}(\text{SolX}) \cdot \text{subs}(c_1 = 1, \text{rhs}(\text{SolT})) \\ & \text{SolGral} := u(x, t) = (c_1 + c_2 e^{\beta^2 x}) e^{-t(\beta^2 - 2)} \end{aligned} \quad (63)$$

$$\begin{aligned} &> \text{Ecua} \\ & \frac{\partial^2}{\partial x^2} u(x, t) + \frac{\partial^2}{\partial t \partial x} u(x, t) = 4 t \left(\frac{\partial}{\partial x} u(x, t) \right) \end{aligned} \quad (64)$$

$$\begin{aligned} &> \text{Comprobar} := \text{simplify}(\text{eval}(\text{subs}(u(x, t) = \text{rhs}(\text{SolGral}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0))) \\ & \text{Comprobar} := 0 = 0 \end{aligned} \quad (65)$$

> restart
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