>

Eamen parcial 2026-1-1
tipo B
Solución

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> *restart*

1) Muestre que la ecuación diferencial dada tiene por solución general presentada & obtenga la solución prticular que satisfaga la condición dada

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> $Ec := y' = \frac{(y+x)}{(y-x)}$

$$Ec := \frac{d}{dx}\, y(x) = \frac{y(x)+x}{y(x)-x} \tag{1}$$

> $SolGral := x^2 - y(x)^2 + 2\cdot x\cdot y(x) = \_C1$

$$SolGral := x^2 - y(x)^2 + 2\,x\,y(x) = \_C1 \tag{2}$$

> $CondIni := y(-2) = 3$

$$CondIni := y(-2) = 3 \tag{3}$$

RESPUESTA

> $DerSolGral := simplify(isolate(diff(SolGral, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\, y(x) = \frac{y(x)+x}{y(x)-x} \tag{4}$$

> $Comprobar := simplify(rhs(Ec) - rhs(DerSolGral) = 0)$

$$Comprobar := 0 = 0 \tag{5}$$

> $Para := subs(x = -2, y(-2) = 3, SolGral)$

$$Para := -17 = \_C1 \tag{6}$$

> $SolPart := subs(\_C1 = lhs(Para), SolGral)$

$$SolPart := x^2 - y(x)^2 + 2\,x\,y(x) = -17 \tag{7}$$

> $DerSolPart := simplify(isolate(diff(SolPart, x), diff(y(x), x)))$

$$DerSolPart := \frac{d}{dx}\, y(x) = \frac{y(x)+x}{y(x)-x} \tag{8}$$

> $ComprobarDos := simplify(rhs(Ec) - rhs(DerSolPart) = 0)$

$$ComprobarDos := 0 = 0 \tag{9}$$

> $ComprobarTres := subs(x = -2, y(-2) = 3, SolPart)$

$$ComprobarTres := -17 = -17 \tag{10}$$

FIN PROBLEMA 1

> *restart*

2) Obtener la solución de la ecuación diferencial dada
Considerar las condiciones iniciales presentadas

> $Ecua := x^2\cdot y' - 2\cdot x\cdot y = x^4\cdot \sin(x)$

$$Ecua := x^2 \left(\frac{d}{dx}\, y(x)\right) - 2\,x\,y(x) = x^4 \sin(x) \tag{11}$$

> $CondIni := y(\text{Pi}) = 3$

$$CondIni := y(\pi) = 3 \tag{12}$$

RESPUESTA

> $EcuaDos := expand\left(\frac{lhs(Ecua)}{x^2}\right) = \frac{rhs(Ecua)}{x^2}$

$$EcuaDos := \frac{d}{dx}\, y(x) - \frac{2\, y(x)}{x} = x^2 \sin(x) \tag{13}$$

> $with(DEtools) :$

> $odeadvisor(EcuaDos)$

$$[\_linear] \tag{14}$$

> $FactInt := intfactor(EcuaDos)$

$$FactInt := \frac{1}{x^2} \tag{15}$$

> $M := -\frac{2\, y}{x} - rhs(EcuaDos)$

$$M := -\frac{2\, y}{x} - x^2 \sin(x) \tag{16}$$

> $N := 1$

$$N := 1 \tag{17}$$

> $MM := expand(FactInt \cdot M)$

$$MM := -\frac{2\, y}{x^3} - \sin(x) \tag{18}$$

> $NN := FactInt \cdot N$

$$NN := \frac{1}{x^2} \tag{19}$$

> $ComprobarUno := diff(MM, y) - diff(NN, x) = 0$

$$ComprobarUno := 0 = 0 \tag{20}$$

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{y}{x^2} + \cos(x) \tag{21}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{y}{x^2} + \cos(x) = \_C1 \tag{22}$$

> $SolGralFinal := expand\left(isolate\left(\frac{y(x)}{x^2} + \cos(x) = \_C1, y(x)\right)\right)$

$$SolGralFinal := y(x) = x^2\, \_C1 - x^2 \cos(x) \tag{23}$$

> $Para := expand(isolate(subs(x = \text{Pi}, rhs(SolGralFinal) = 3), \_C1))$

$$Para := \_C1 = -1 + \frac{3}{\pi^2} \tag{24}$$

> $SolPartFinal := expand(subs(\_C1 = rhs(Para), SolGralFinal))$

$$SolPartFinal := y(x) = -x^2 + \frac{3\,x^2}{\pi^2} - x^2\cos(x) \tag{25}$$

> $ComprobarDos := simplify(subs(x = \mathrm{Pi}, y(\mathrm{Pi}) = 3, SolPartFinal))$

$$ComprobarDos := 3 = 3 \tag{26}$$

FIN PROBLEMA 2)

> $restart$

3) Obtenga la solución general de la ecuación diferencial

>

> $Ecua := x\cdot y' - y = x\cdot \sec\left(\frac{y}{x}\right)$

$$Ecua := x\left(\frac{d}{dx}\,y(x)\right) - y(x) = x\sec\left(\frac{y(x)}{x}\right) \tag{27}$$

RESPUESTA

> $EcuaDos := expand\left(\frac{lhs(Ecua)}{x}\right) = \frac{rhs(Ecua)}{x}$

$$EcuaDos := \frac{d}{dx}\,y(x) - \frac{y(x)}{x} = \sec\left(\frac{y(x)}{x}\right) \tag{28}$$

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[[\_homogeneous, class\ A], \_dAlembert] \tag{29}$$

> $EcuaTres := eval(subs(y(x) = x\cdot u(x), EcuaDos))$

$$EcuaTres := x\left(\frac{d}{dx}\,u(x)\right) = \sec(u(x)) \tag{30}$$

> $EcuaCuatro := lhs(EcuaTres) - rhs(EcuaTres) = 0$

$$EcuaCuatro := x\left(\frac{d}{dx}\,u(x)\right) - \sec(u(x)) = 0 \tag{31}$$

> $P := -1;\ Q := \sec(u);\ R := x;\ S := 1$

$$P := -1$$
$$Q := \sec(u)$$
$$R := x$$
$$S := 1 \tag{32}$$

> $SolGralCero := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, u\right) = \_C1$

$$SolGralCero := -\ln(x) + \sin(u) = \_C1 \tag{33}$$

> $SolGral := subs\left(u = \frac{y(x)}{x}, SolGralCero\right)$

$$SolGral := -\ln(x) + \sin\left(\frac{y(x)}{x}\right) = \_C1 \tag{34}$$

> $DerSolGral := simplify(expand(isolate(diff(SolGral, x), diff(y(x), x))))$

(35)

$$DerSolGral := \frac{d}{dx}\, y(x) = \sec\left(\frac{y(x)}{x}\right) + \frac{y(x)}{x} \tag{35}$$

> $DerEcua := expand(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}\, y(x) = \sec\left(\frac{y(x)}{x}\right) + \frac{y(x)}{x} \tag{36}$$

> $Comprobar := rhs(DerSolGral) - rhs(DerEcua) = 0$

$$Comprobar := 0 = 0 \tag{37}$$

FIN PROBLEMA 3

> $restart$

4) Obtener la solución general de la ecuación diferencial no homogénea dada
Si las soluciones presentes resuelven la ecuación diferencial homogénea asociada a la dada

>

> $Ecua := x \cdot y'' + (1 - 2 \cdot x) \cdot y' + (x - 1) \cdot y = x \cdot \exp(x)$

$$Ecua := x\left(\frac{d^2}{dx^2}\, y(x)\right) + (1 - 2\,x)\left(\frac{d}{dx}\, y(x)\right) + (x - 1)\, y(x) = x\, e^x \tag{38}$$

> $yy[1] := \exp(x);\ yy[2] := \exp(x) \cdot \log(x)$

$$yy_1 := e^x$$

$$yy_2 := e^x \ln(x) \tag{39}$$

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := x\left(\frac{d^2}{dx^2}\, y(x)\right) + (1 - 2\,x)\left(\frac{d}{dx}\, y(x)\right) + (x - 1)\, y(x) = 0 \tag{40}$$

RESPUESTA

> $EcuaDosNoHom := expand\left(\frac{lhs(Ecua)}{x}\right) = \frac{rhs(Ecua)}{x}$

$$EcuaDosNoHom := \frac{d^2}{dx^2}\, y(x) + \frac{\frac{d}{dx}\, y(x)}{x} - 2\,\frac{d}{dx}\, y(x) + y(x) - \frac{y(x)}{x} = e^x \tag{41}$$

> $EcuaDosHom := expand\left(\frac{lhs(EcuaHom)}{x}\right) = 0$

$$EcuaDosHom := \frac{d^2}{dx^2}\, y(x) + \frac{\frac{d}{dx}\, y(x)}{x} - 2\,\frac{d}{dx}\, y(x) + y(x) - \frac{y(x)}{x} = 0 \tag{42}$$

> $SolHom := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolHom := y(x) = \_C1\, e^x + \_C2\, e^x \ln(x) \tag{43}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolHom), EcuaDosHom)))$

$$Comprobar := 0 = 0 \tag{44}$$

> $Q := rhs(EcuaDosNoHom)$

$$Q := e^x \tag{45}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^x & e^x \ln(x) \\ e^x & e^x \ln(x) + \frac{e^x}{x} \end{bmatrix} \quad \textbf{(46)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & e^x \end{bmatrix} \quad \textbf{(47)}$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} -\ln(x)\, x & x \end{bmatrix} \quad \textbf{(48)}$$

> $Aprima := ParaVar[1]$

$$Aprima := -\ln(x)\, x \quad \textbf{(49)}$$

> $Bprima := ParaVar[2]$

$$Bprima := x \quad \textbf{(50)}$$

> $SolGralNoHom := y(x) = expand((int(Aprima, x) + \_C1) \cdot yy[1] + (int(Bprima, x) + \_C2) \cdot yy[2])$

$$SolGralNoHom := y(x) = \frac{e^x x^2}{4} + \_C1\, e^x + \_C2\, e^x \ln(x) \quad \textbf{(51)}$$

> $ComprobarDos := simplify(eval(subs(y(x) = rhs(SolGralNoHom), EcuaDosNoHom)))$

$$ComprobarDos := e^x = e^x \quad \textbf{(52)}$$

FIN PROBLEMA 4

> $restart$

5) Determine la solución general de la ecuación diferencia

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> $Ecua := y'' + 2 \cdot y' + 2 \cdot y = \exp(-x) \cdot \sec(x)$

$$Ecua := \frac{d^2}{dx^2} y(x) + 2 \frac{d}{dx} y(x) + 2\, y(x) = e^{-x} \sec(x) \quad \textbf{(53)}$$

RESPUESTA

> $EcuaHom := lhs(Ecua) = 0$

$$EcuaHom := \frac{d^2}{dx^2} y(x) + 2 \frac{d}{dx} y(x) + 2\, y(x) = 0 \quad \textbf{(54)}$$

> $EcuaCarac := m^2 + 2 \cdot m + 2 = 0$

$$EcuaCarac := m^2 + 2\, m + 2 = 0 \quad \textbf{(55)}$$

> $Q := rhs(Ecua)$

$$Q := e^{-x} \sec(x) \quad \textbf{(56)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := -1 + I, -1 - I \quad \textbf{(57)}$$

> $yy[1] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \cos(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_1 := e^{-x} \cos(x) \quad \textbf{(58)}$$

> $yy[2] := \exp(\mathrm{Re}(Raiz[1]) \cdot x) \cdot \sin(\mathrm{Im}(Raiz[1]) \cdot x)$

$$yy_2 := e^{-x}\sin(x) \tag{59}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{-x}\cos(x) & e^{-x}\sin(x) \\ -e^{-x}\cos(x) - e^{-x}\sin(x) & -e^{-x}\sin(x) + e^{-x}\cos(x) \end{bmatrix} \tag{60}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & e^{-x}\sec(x) \end{bmatrix} \tag{61}$$

> $ParaVar := simplify(linsolve(WW, BB))$

$$ParaVar := \begin{bmatrix} -\tan(x) & 1 \end{bmatrix} \tag{62}$$

> $Aprima := ParaVar[1]$

$$Aprima := -\tan(x) \tag{63}$$

> $Bprima := ParaVar[2]$

$$Bprima := 1 \tag{64}$$

> $SolGralNoHom := y(x) = expand(simplify((int(Aprima, x) + \_C1)\cdot yy[1] + (int(Bprima, x) + \_C2)\cdot yy[2]))$

$$SolGralNoHom := y(x) = \frac{\sin(x)\ \_C2}{e^{x}} + \frac{\sin(x)\ x}{e^{x}} + \frac{\cos(x)\ \ln(\cos(x))}{e^{x}} + \frac{\cos(x)\ \_C1}{e^{x}} \tag{65}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolGralNoHom), Ecua)))$

$$Comprobar := e^{-x}\sec(x) = e^{-x}\sec(x) \tag{66}$$

FIN PROBLEMA 5

> $restart$

FIN EXAMEN 2026-1-1 TIPO B

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