> *restart*

SEGUNDO EXAMEN FINAL

> *restart*

1)

> $(x^4 \cdot \log(x) - 2 \cdot x \cdot y^3) + 3 \cdot x^2 \cdot y^2 \cdot y' = 0; y(2) = 2$

$$x^4 \ln(x) - 2\, x\, y(x)^3 + 3\, x^2\, y(x)^2 \left(\frac{d}{dx}\, y(x)\right) = 0$$

$$y(2) = 2 \quad \textbf{(1)}$$

> *restart*

Respuesta 1

> $Ecua := (x^4 \cdot \log(x) - 2 \cdot x \cdot y^3) + 3 \cdot x^2 \cdot y^2 \cdot y' = 0$

$$Ecua := x^4 \ln(x) - 2\, x\, y(x)^3 + 3\, x^2\, y(x)^2 \left(\frac{d}{dx}\, y(x)\right) = 0 \quad \textbf{(2)}$$

> $CondIni := y(2) = 2$

$$CondIni := y(2) = 2 \quad \textbf{(3)}$$

> $with(DEtools):$

> $odeadvisor(Ecua)$

$$[\_Bernoulli] \quad \textbf{(4)}$$

> $FacInt := intfactor(Ecua)$

$$FacInt := \frac{1}{x^4} \quad \textbf{(5)}$$

> $M := (x^4 \cdot \log(x) - 2 \cdot x \cdot y^3)$

$$M := x^4 \ln(x) - 2\, x\, y^3 \quad \textbf{(6)}$$

> $N := 3 \cdot x^2 \cdot y^2$

$$N := 3\, x^2\, y^2 \quad \textbf{(7)}$$

> $MM := expand(FacInt \cdot M)$

$$MM := \ln(x) - \frac{2\, y^3}{x^3} \quad \textbf{(8)}$$

> $NN := expand(FacInt \cdot N)$

$$NN := \frac{3\, y^2}{x^2} \quad \textbf{(9)}$$

> $Comprobar := diff(MM, y) = diff(NN, x)$

$$Comprobar := -\frac{6\, y^2}{x^3} = -\frac{6\, y^2}{x^3} \quad \textbf{(10)}$$

Es exacta

> $IntMMx := int(MM, x)$

$$IntMMx := \frac{y^3}{x^2} + x \ln(x) - x \quad \textbf{(11)}$$

> $SolGral := IntMMx + int((NN - diff(IntMMx, y)), y) = \_C1$

$$SolGral := \frac{y^3}{x^2} + x\ln(x) - x = \_C1 \quad \textbf{(12)}$$

> $SolGralDos := \frac{y(x)^3}{x^2} + x\ln(x) - x = \_C1$

$$SolGralDos := \frac{y(x)^3}{x^2} + x\ln(x) - x = \_C1 \quad \textbf{(13)}$$

> $DerSolGral := expand(isolate(diff(SolGralDos, x), diff(y(x), x)))$

$$DerSolGral := \frac{d}{dx}\, y(x) = \frac{2\, y(x)}{3\, x} - \frac{x^2 \ln(x)}{3\, y(x)^2} \quad \textbf{(14)}$$

> $DerEcua := expand(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}\, y(x) = \frac{2\, y(x)}{3\, x} - \frac{x^2 \ln(x)}{3\, y(x)^2} \quad \textbf{(15)}$$

> $Para := simplify(subs(y = 2, x = 2, SolGral))$

$$Para := 2\ln(2) = \_C1 \quad \textbf{(16)}$$

> $SolPart := subs(\_C1 = lhs(Para), SolGralDos)$

$$SolPart := \frac{y(x)^3}{x^2} + x\ln(x) - x = 2\ln(2) \quad \textbf{(17)}$$

> restart

2)

> $(x\cdot y^2 - y^2 + x - 1) + (x^2\cdot y - 2\cdot x\cdot y + x^2 + 2\cdot y - 2x + 2)\cdot y' = 0$

$$x\, y(x)^2 - y(x)^2 + x - 1 + (x^2 y(x) - 2x\, y(x) + x^2 + 2\, y(x) - 2x + 2)\left(\frac{d}{dx}\, y(x)\right) = 0 \quad \textbf{(18)}$$

> restart

RESPUESTA 2

> $Ecua := (x\cdot y^2 - y^2 + x - 1) + (x^2\cdot y - 2\cdot x\cdot y + x^2 + 2\cdot y - 2x + 2)\cdot y' = 0$

$$Ecua := x\, y(x)^2 - y(x)^2 + x - 1 + (x^2 y(x) - 2x\, y(x) + x^2 + 2\, y(x) - 2x + 2)\left(\frac{d}{dx}\, y(x)\right) = 0 \quad \textbf{(19)}$$

> with(DEtools) :

> odeadvisor(Ecua)

$$[\_separable] \quad \textbf{(20)}$$

> $M := simplify((x\cdot y^2 - y^2 + x - 1))$

$$M := (y^2 + 1)(-1 + x) \quad \textbf{(21)}$$

> $N := simplify((x^2\cdot y - 2\cdot x\cdot y + x^2 + 2\cdot y - 2x + 2))$

$$N := (x^2 - 2x + 2)(y + 1) \quad \textbf{(22)}$$

> $P := x - 1; Q := (y^2 + 1); R := (x^2 - 2x + 2); S := (y + 1)$

$$P := -1 + x$$

$$Q := y^2 + 1$$

$$R := x^2 - 2x + 2$$
$$S := y + 1 \quad \textbf{(23)}$$

> $SolGral := int\left(\frac{P}{R}, x\right) + int\left(\frac{S}{Q}, y\right) = \_C1$

$$SolGral := \frac{\ln(x^2 - 2x + 2)}{2} + \frac{\ln(y^2 + 1)}{2} + \arctan(y) = \_C1 \quad \textbf{(24)}$$

> $SolFinal := \frac{\ln(x^2 - 2x + 2)}{2} + \frac{\ln(y(x)^2 + 1)}{2} + \arctan(y(x)) = \_C1$

$$SolFinal := \frac{\ln(x^2 - 2x + 2)}{2} + \frac{\ln(y(x)^2 + 1)}{2} + \arctan(y(x)) = \_C1 \quad \textbf{(25)}$$

> $DerSolFinal := simplify(isolate(diff(SolFinal, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\, y(x) = \frac{(1 - x)\,(y(x)^2 + 1)}{(x^2 - 2x + 2)\,(y(x) + 1)} \quad \textbf{(26)}$$

> $DerEcua := factor(isolate(Ecua, diff(y(x), x)))$

$$DerEcua := \frac{d}{dx}\, y(x) = -\frac{(-1 + x)\,(y(x)^2 + 1)}{(x^2 - 2x + 2)\,(y(x) + 1)} \quad \textbf{(27)}$$

> $Comprobar := simplify(rhs(DerSolFinal) - rhs(DerEcua) = 0)$

$$Comprobar := 0 = 0 \quad \textbf{(28)}$$

> $restart$

3)

> $y'' - 8 \cdot y' + 16 \cdot y = (1 - x) \cdot \exp(4x)$

$$\frac{d^2}{dx^2}\, y(x) - 8\, \frac{d}{dx}\, y(x) + 16\, y(x) = (1 - x)\, e^{4x} \quad \textbf{(29)}$$

> $restart$

RESPUESTA 3

> $Ecua := y'' - 8 \cdot y' + 16 \cdot y = (1 - x) \cdot \exp(4x)$

$$Ecua := \frac{d^2}{dx^2}\, y(x) - 8\, \frac{d}{dx}\, y(x) + 16\, y(x) = (1 - x)\, e^{4x} \quad \textbf{(30)}$$

> $EcuaHom := lhs(Ecua)$

$$EcuaHom := \frac{d^2}{dx^2}\, y(x) - 8\, \frac{d}{dx}\, y(x) + 16\, y(x) \quad \textbf{(31)}$$

> $Q := rhs(Ecua)$

$$Q := (1 - x)\, e^{4x} \quad \textbf{(32)}$$

> $EcuaCarac := m^2 - 8 \cdot m + 16 = 0$

$$EcuaCarac := m^2 - 8\, m + 16 = 0 \quad \textbf{(33)}$$

> $Raiz := solve(EcuaCarac)$

$$Raiz := 4, 4 \quad \textbf{(34)}$$

> $yy[1] := \exp(4x);\ yy[2] := x \cdot \exp(4x)$

$$yy_1 := e^{4x}$$
$$yy_2 := x\,e^{4x} \quad \textbf{(35)}$$

> $with(linalg):$

> $WW := wronskian([yy[1], yy[2]], x)$

$$WW := \begin{bmatrix} e^{4x} & x\,e^{4x} \\ 4\,e^{4x} & e^{4x} + 4\,x\,e^{4x} \end{bmatrix} \quad \textbf{(36)}$$

> $BB := array([0, Q])$

$$BB := \begin{bmatrix} 0 & (1-x)\,e^{4x} \end{bmatrix} \quad \textbf{(37)}$$

> $ParaVar := linsolve(WW, BB)$

$$ParaVar := \begin{bmatrix} x\,(x-1) & 1-x \end{bmatrix} \quad \textbf{(38)}$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := x\,(x-1)$$
$$Bprima := 1-x \quad \textbf{(39)}$$

> $SolPart := y(x) = simplify(int(Aprima, x) \cdot yy[1] + int(Bprima, x) \cdot yy[2])$

$$SolPart := y(x) = -\frac{e^{4x}\,x^2\,(x-3)}{6} \quad \textbf{(40)}$$

> $SolGral := y(x) = \_C1 \cdot yy[1] + \_C2 \cdot yy[2]$

$$SolGral := y(x) = \_C1\,e^{4x} + \_C2\,x\,e^{4x} \quad \textbf{(41)}$$

> $SolFinal := y(x) = rhs(SolGral) + rhs(SolPart)$

$$SolFinal := y(x) = \_C1\,e^{4x} + \_C2\,x\,e^{4x} - \frac{e^{4x}\,x^2\,(x-3)}{6} \quad \textbf{(42)}$$

> $Ecua$

$$\frac{d^2}{dx^2}\,y(x) - 8\,\frac{d}{dx}\,y(x) + 16\,y(x) = (1-x)\,e^{4x} \quad \textbf{(43)}$$

> $Comprobar := simplify(eval(subs(y(x) = rhs(SolFinal), lhs(Ecua) - rhs(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad \textbf{(44)}$$

> $restart$

4)

> $diff(x(t), t\$2) + 3 \cdot diff(x(t), t) + 2 \cdot x = 2 \cdot t^2 + 1; x(0) = 4; D(x)(0) = -3$

$$\frac{d^2}{dt^2}\,x(t) + 3\,\frac{d}{dt}\,x(t) + 2\,x = 2\,t^2 + 1$$
$$x(0) = 4$$
$$D(x)(0) = -3 \quad \textbf{(45)}$$

> $restart$

RESPUESTA 4)

> $Ecua := diff(x(t), t\$2) + 3 \cdot diff(x(t), t) + 2 \cdot x(t) = 2 \cdot t^2 + 1$

(46)

$$Ecua := \frac{d^2}{dt^2}\, x(t) + 3\,\frac{d}{dt}\, x(t) + 2\,x(t) = 2\,t^2 + 1 \quad (46)$$

> $CondIni := x(0) = 4, D(x)(0) = -3$

$$CondIni := x(0) = 4, D(x)(0) = -3 \quad (47)$$

> $with(inttrans):$

> $EcuaTL := subs(CondIni, laplace(Ecua, t, s))$

$$EcuaTL := s^2\,\mathcal{L}(x(t), t, s) - 9 - 4\,s + 3\,s\,\mathcal{L}(x(t), t, s) + 2\,\mathcal{L}(x(t), t, s) = \frac{4}{s^3} + \frac{1}{s} \quad (48)$$

> $SolTL := simplify(isolate(EcuaTL, laplace(x(t), t, s)))$

$$SolTL := \mathcal{L}(x(t), t, s) = \frac{4\,s^2 - 3\,s + 2}{s^3} \quad (49)$$

> $SolPart := invlaplace(SolTL, s, t)$

$$SolPart := x(t) = t^2 - 3\,t + 4 \quad (50)$$

> $Ecua$

$$\frac{d^2}{dt^2}\, x(t) + 3\,\frac{d}{dt}\, x(t) + 2\,x(t) = 2\,t^2 + 1 \quad (51)$$

> $Comprobar := eval(subs(x(t) = rhs(SolPart), Ecua))$

$$Comprobar := 2\,t^2 + 1 = 2\,t^2 + 1 \quad (52)$$

> $restart$

5)

> $diff(x(t), t) + x(t) = y(t) + \exp(t); diff(y(t), t) + y(t) = x(t) + \exp(t); x(0) = 1; y(0) = 1$

$$\frac{d}{dt}\, x(t) + x(t) = y(t) + e^t$$

$$\frac{d}{dt}\, y(t) + y(t) = x(t) + e^t$$

$$x(0) = 1$$

$$y(0) = 1 \quad (53)$$

>

> $restart$

RESPUESTA 5

> $Sistema := diff(x(t), t) + x(t) = y(t) + \exp(t), diff(y(t), t) + y(t) = x(t) + \exp(t):$
> $Sistema[1]; Sistema[2]$

$$\frac{d}{dt}\, x(t) + x(t) = y(t) + e^t$$

$$\frac{d}{dt}\, y(t) + y(t) = x(t) + e^t \quad (54)$$

> $CondIni := x(0) = 1, y(0) = 1$

$$CondIni := x(0) = 1, y(0) = 1 \quad (55)$$

> $AA := array([[-1, 1], [1, -1]])$

$$AA := \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad (56)$$

> $XYcero := array([1, 1])$

$$XYcero := \begin{bmatrix} 1 & 1 \end{bmatrix} \quad (57)$$

> $BB := array([\exp(t), \exp(t)])$

$$BB := \begin{bmatrix} e^t & e^t \end{bmatrix} \quad (58)$$

> $with(linalg):$

> $MatExp := exponential(AA, t)$

$$MatExp := \begin{bmatrix} \frac{e^{-2t}}{2} + \frac{1}{2} & \frac{1}{2} - \frac{e^{-2t}}{2} \\ \frac{1}{2} - \frac{e^{-2t}}{2} & \frac{e^{-2t}}{2} + \frac{1}{2} \end{bmatrix} \quad (59)$$

> $SolGralNoHom := evalm(MatExp \&* XYcero) : x(t) = SolGralNoHom[1]; y(t) = SolGralNoHom[2]$

$$x(t) = 1$$
$$y(t) = 1 \quad (60)$$

> $MatExpTau := map(rcurry(eval, t = 't - \text{tau}'), MatExp)$

$$MatExpTau := \begin{bmatrix} \frac{e^{-2t+2\tau}}{2} + \frac{1}{2} & \frac{1}{2} - \frac{e^{-2t+2\tau}}{2} \\ \frac{1}{2} - \frac{e^{-2t+2\tau}}{2} & \frac{e^{-2t+2\tau}}{2} + \frac{1}{2} \end{bmatrix} \quad (61)$$

> $BBtau := map(rcurry(eval, t = '\text{tau}'), BB)$

$$BBtau := \begin{bmatrix} e^\tau & e^\tau \end{bmatrix} \quad (62)$$

> $ProdTau := evalm(MatExpTau \&* BBtau)$

$$ProdTau := \left[\left(\frac{e^{-2t+2\tau}}{2} + \frac{1}{2}\right) e^\tau + \left(\frac{1}{2} - \frac{e^{-2t+2\tau}}{2}\right) e^\tau, \left(\frac{e^{-2t+2\tau}}{2} + \frac{1}{2}\right) e^\tau + \left(\frac{1}{2} - \frac{e^{-2t+2\tau}}{2}\right) e^\tau \right] \quad (63)$$

> $SolPart := map(int, ProdTau, \text{tau} = 0..t) : x(t) = SolPart[1]; y(t) = SolPart[2]$

$$x(t) = -1 + e^t$$
$$y(t) = -1 + e^t \quad (64)$$

> $SolFinal := SolGralNoHom[1] + SolPart[1], SolGralNoHom[2] + SolPart[2] : x(t) = SolFinal[1]; y(t) = SolFinal[2]$

$$x(t) = e^t$$
$$y(t) = e^t \quad (65)$$

> $ComprobarUno := eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2], lhs(Sistema[1]) - rhs(Sistema[1]) = 0))$

$$ComprobarUno := 0 = 0 \quad (66)$$

> $ComprobarDos := eval(subs(x(t) = SolFinal[1], y(t) = SolFinal[2], lhs(Sistema[2]) - rhs(Sistema[2]) = 0))$

$$ComprobarDos := 0 = 0 \quad (67)$$

>

> restart

6) para constante de separacion negativa

> $diff(u(x, t), t\$3) - 4 \cdot diff(u(x, t), t, x) = 0$

$$\frac{\partial^3}{\partial t^3} u(x, t) - 4 \frac{\partial^2}{\partial t \partial x} u(x, t) = 0 \quad (68)$$

> restart

RESPUESTA 6

> $Ecua := diff(u(x, t), t\$3) - 4 \cdot diff(u(x, t), t, x) = 0$

$$Ecua := \frac{\partial^3}{\partial t^3} u(x, t) - 4 \frac{\partial^2}{\partial t \partial x} u(x, t) = 0 \quad (69)$$

> $EcuaDos := eval(subs(u(x, t) = F(x) \cdot G(t), Ecua))$

$$EcuaDos := F(x) \left(\frac{d^3}{dt^3} G(t) \right) - 4 \left(\frac{d}{dx} F(x) \right) \left(\frac{d}{dt} G(t) \right) = 0 \quad (70)$$

> $EcuaTres := lhs(EcuaDos) + 4 \left(\frac{d}{dx} F(x) \right) \left(\frac{d}{dt} G(t) \right) = rhs(EcuaDos) + 4 \left(\frac{d}{dx} F(x) \right) \left(\frac{d}{dt} G(t) \right)$

$$EcuaTres := F(x) \left(\frac{d^3}{dt^3} G(t) \right) = 4 \left(\frac{d}{dx} F(x) \right) \left(\frac{d}{dt} G(t) \right) \quad (71)$$

> $EcuaSep := \frac{lhs(EcuaTres)}{4 \cdot F(x) \cdot \left(\frac{d}{dt} G(t) \right)} = \frac{rhs(EcuaTres)}{4 \cdot F(x) \cdot \left(\frac{d}{dt} G(t) \right)}$

$$EcuaSep := \frac{\frac{d^3}{dt^3} G(t)}{4 \left(\frac{d}{dt} G(t) \right)} = \frac{\frac{d}{dx} F(x)}{F(x)} \quad (72)$$

> $EcuaF := rhs(EcuaSep) = -\beta^2;\ EcuaG := lhs(EcuaSep) = -\beta^2$

$$EcuaF := \frac{\frac{d}{dx} F(x)}{F(x)} = -\beta^2$$

$$EcuaG := \frac{\frac{d^3}{dt^3} G(t)}{4 \left(\frac{d}{dt} G(t) \right)} = -\beta^2 \quad (73)$$

> $SolF := dsolve(EcuaF); SolG := dsolve(EcuaG)$

$$SolF := F(x) = c_1 \, e^{-\beta^2 x}$$

$$SolG := G(t) = c_1 + c_2 \sin(2 \beta t) + c_3 \cos(2 \beta t) \quad \textbf{(74)}$$

> $SolFinal := u(x, t) = subs(c_1 = 1, rhs(SolF)) \cdot rhs(SolG)$

$$SolFinal := u(x, t) = e^{-\beta^2 x} \left(c_1 + c_2 \sin(2 \beta t) + c_3 \cos(2 \beta t) \right) \quad \textbf{(75)}$$

> $Comprobar := simplify(eval(subs(u(x, t) = rhs(SolFinal), Ecua)))$

$$Comprobar := 0 = 0 \quad \textbf{(76)}$$

> restart

FIN DEL EXAMEN

>

>